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Mapping Conic Sections and Estimating Surface Response using Non-Response Analysis

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Editorial

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NON-RESPONSE ANALYSIS AND CONIC SECTIONS

Non-Response analysis uses ordinary least squares and maximum likelihood estimates to fit co-dependent variables in an implicit form including conic sections such as circles, ellipse, hyperbolas, and spheres, among other higher dimensional surfaces.

Conic sections in 2-dimensions are relationships between two measures that take the form

$$Ax^2 + Bx + Cxy + Dy + Ey^2 + Fxy = G.$$

This form includes lines, circles, ellipse, parabolas and hyperbolas [1,2].

Consider an example in a conical relationship:

$13x^2$-10xy+$13y^2$=72.

Simulation of an Ellipse

To simulate an ellipse, let time 't', be taken in random intervals over one unit of measurement,

t ∈ (0,1).

To simulate an ellipse, consider the transformed space (*u, v*) such that in the first direction, the measure *u* moves linearly over time, either increasing or decreasing at a rate of 3 units per unit measure of time, *u*=± 3t; and in the second direction, the measure v moves along one of two curves, $v = \pm 2\sqrt{1-t^2}$. Further transformation of the (*u, v*) space to the (*x, y*) space by rotating the resulting ellipse 45° counterclockwise, letting $x = \frac{\sqrt{2}}{2}(u-v)$ and $y = \frac{\sqrt{2}}{2}(u+v)$. Random error exists in both variables: $x_i = x + N(0,\sigma^2)$ and $y_i = y + N(0,\sigma^2)$; with a sample of size 100, **(Figure 1).**

Using standard regression on the full second-order model as given by,

$$y = \beta_0 + \beta_1 x + \beta_2 x^2,$$

Which only accounts for the central tendency and does not allow for the subject response to be conditional, **(Figure 2a).** The model can be improved slightly if we partition the data and fit points above this nearly linear equation (d =1) separately from those below the developed equation (d =1); that is,

$$y = \begin{cases} \beta_{00} + \beta_{01}x + \beta_{02}x^2 & d = 0 \\ \beta_{10} + \beta_{11}x + \beta_{21}x^2 & d = 1 \end{cases},$$

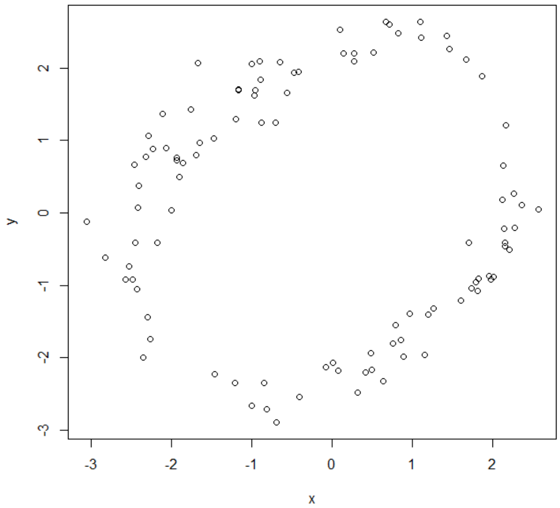

Figure 1. Scatter plot of simulated data with $\sigma = 0.25$.

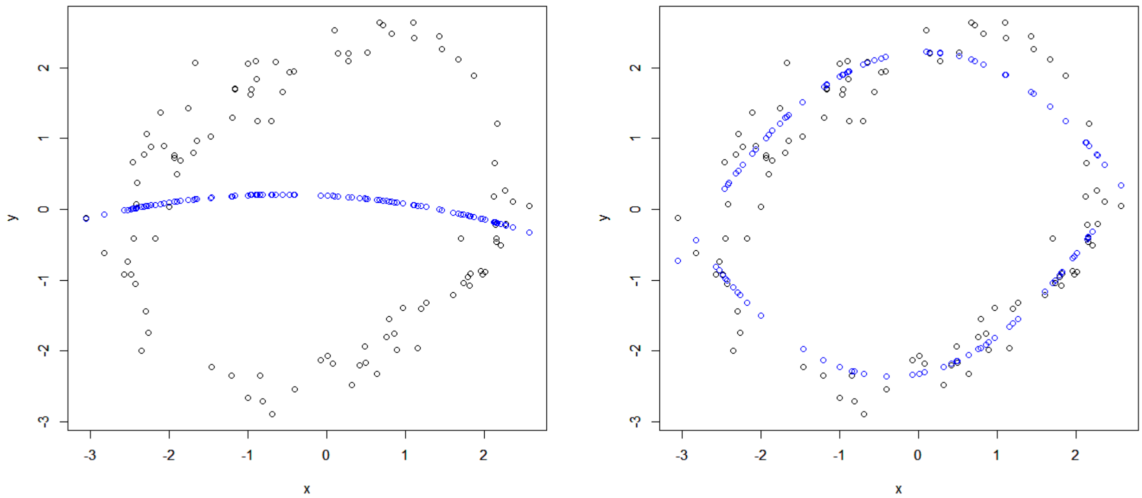

Figure 2. Scatter plot of full second-order model using standard regression; (a) using the entire data set and (b) partitioning the data based on the first developed model.

This gives better point estimates then the single estimate, **(Figure 2b).**

Using non-response analysis on the full second-order interactive model,

$$1 = \alpha_1 x + \alpha_2 y + \alpha_3 x^2 + \alpha_4 y^2 + \alpha_5 xy,$$

And solving for y using the quadratic equation yields much better results, **(Figure 3)**;

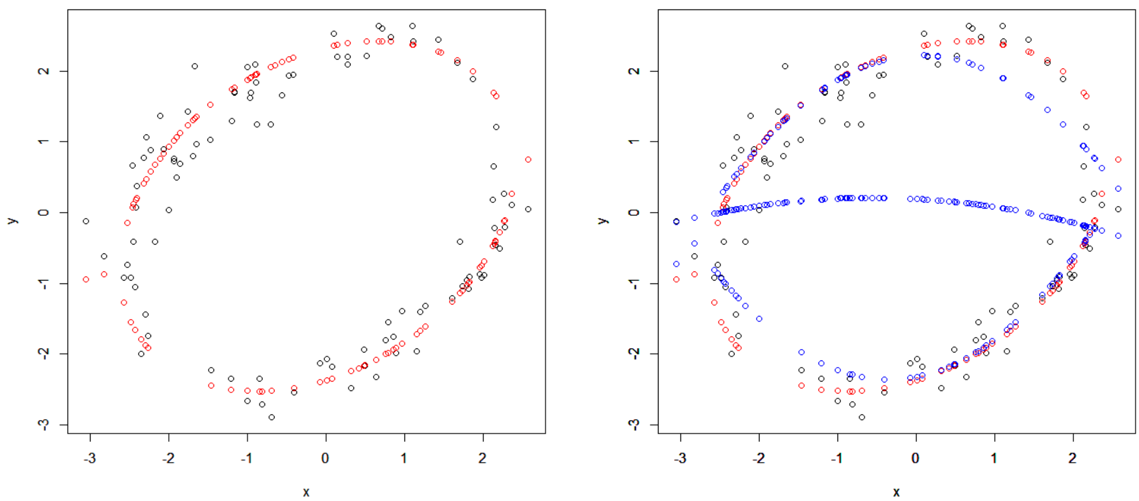

Figure 3. Scatter plot of (a) full second-order interactive model using non-response analysis and (b) a comparison of non-response analysis (red) with the standard methods (blue).

$$\hat{y} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

Where, $a = \hat{\alpha}_4$, $b = \hat{\alpha}_2 + \hat{\alpha}_5 x$ and $c = \left(\hat{\alpha}_1 x + \hat{\alpha}_3 x^2\right) - 1$. Note: if $d = b^2 - 4ac < 0$, then $\hat{y} = \frac{-b}{2a} = \frac{\hat{\alpha}_2 + \hat{\alpha}_5 x}{\hat{\alpha}_4}$, a line as shown outside of the ellipse in **Figure 3a.**

Compare this with the model full second-order model first for the data overall and using the partitioned data, **(Figure 3b).**

Non-response Analysis in Three-Dimensions

Non-response analysis can also model in multi-dimensional space and is only restricted by the tractability of the variable measures [3].

Non-response Analysis on Spheres

Consider more complex models such as data which lies in a spherical relationship that is observed over time. For example, consider the relationship between three variables, one considered to be the subject response, *y*, and the other two explanatory: *x* and *z*; and the true state of nature be outlined by,

$$\left(x-h\right)^2 + \left(y-k\right)^2 + \left(z-l\right)^2 = r^2.$$

For a sphere with fixed center (*h, k, l*) and fixed radius *r*.

To simulate such data, let the three measured variables be such that without random error, they are points on a sphere as given by

$$x = r\cos\left(f_1 t\right)\sin\left(f_2 t\right) + h,$$
$$y = r\sin\left(f_1 t\right)\sin\left(f_2 t\right) + k$$
$$z = r\cos\left(f_2 t\right) + l,$$

where the time measured in random intervals for multiple cycles $t \sim U[0, 8\pi]$; and the three random error terms are normally distributed with a mean of zero and constant variance: that is τ, δ, and ε are $N(0, \sigma^2)$.

$$x_i = r\cos\left(f_1 t_i\right)\sin\left(f_2 t_i\right) + h + \tau_i,$$
$$y = r\sin\left(f_1 t_i\right)\sin\left(f_2 t_i\right) + k + \varepsilon_i$$
$$z = r\cos\left(f_2 t_i\right) + l + \delta,$$

Just looking at the data, there is no apparent relationship other than the somewhat circular nature of the data **(Figure 4).**

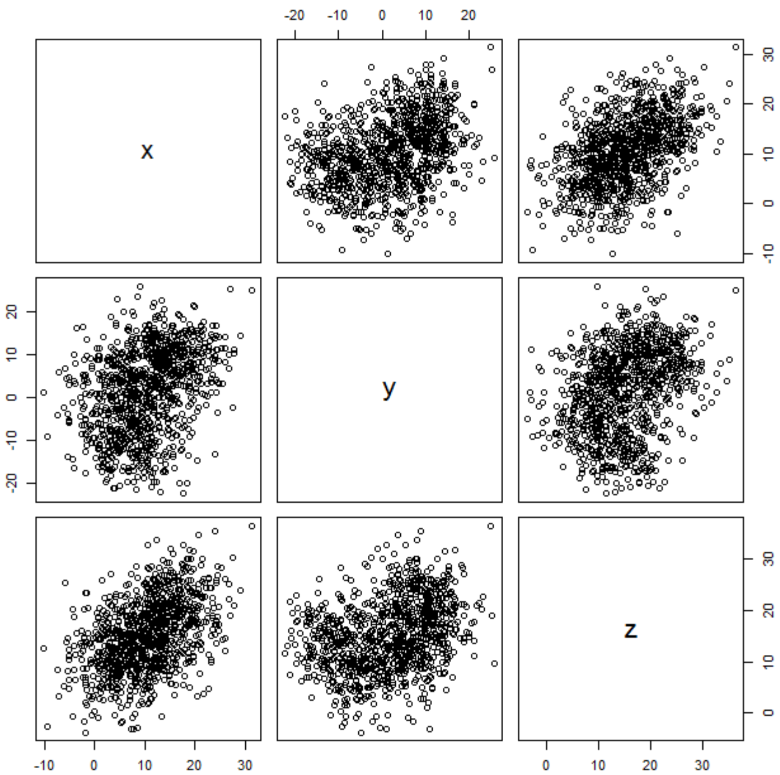

Figure 4. Scatter plots of simulated data shown in pairs.

Using regression on the full second-order interactive model,

$$y = \beta_0 + \beta_1 x + \beta_2 z + \beta_3 x^2 + \beta_4 z^2 + \beta_5 xz,$$

This mainly accounts for the central tendency and does not allow for higher order terms and interaction with the subject response **(Figure 5).**

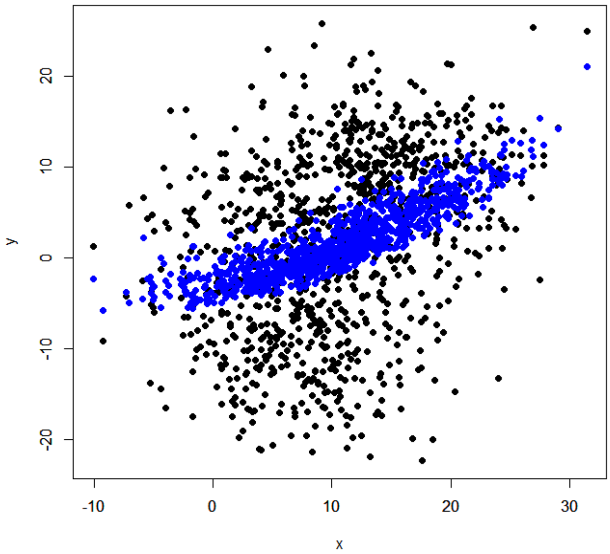

Figure 5. Scatter plot of full second-order interactive model using standard regression.

Using non-response analysis on the full second-order interactive model

$$1 = \alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_4 x^2 + \alpha_5 y^2 + \alpha_6 z^2 + \alpha_7 xy + \alpha_8 xz + \alpha_9 yz,$$

And solving for y using the quadratic equation yields much better results, **(Figure 6)**;

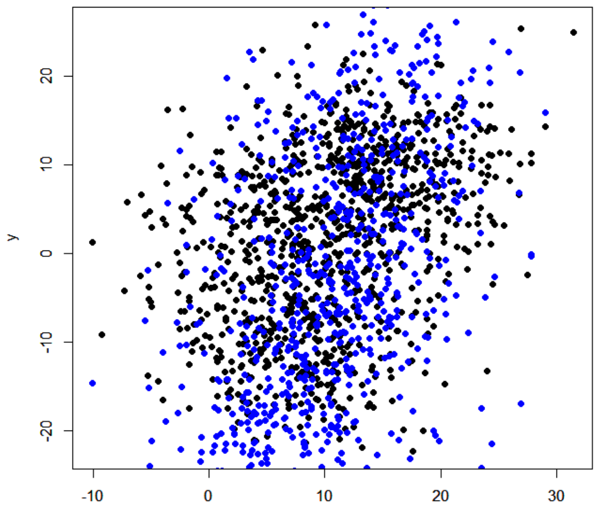

Figure 6. Scatter plot of full second-order interactive model using non-response analysis.

$$\hat{y} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

Where $a = \hat{\alpha}_5$, $b = \hat{\alpha}_2 + \hat{\alpha}_7 x + \hat{\alpha}_9 z$, and $c = \left(\hat{\alpha}_1 x + \hat{\alpha}_3 z + \hat{\alpha}_4 x^2 + \hat{\alpha}_6 z^2 + \hat{\alpha}_8 xz\right) - 1$.

Consider the space of a sphere which is transforming over time as the radius oscillates between one and three [4,5].

Let time be randomly observed over a period of 8π, and three measures be such that $x^2 + y^2 + z^2 = r^2$, where $r(t)$ is an unknown measure but is believe to change over time. For simulation purposes, the radius will be defined to be between one and three with a period of 5π. Furthermore, let each measured variable include random error that is normally distributed with expected value of zero and equal constant variance **(Figures 7 and 8).**

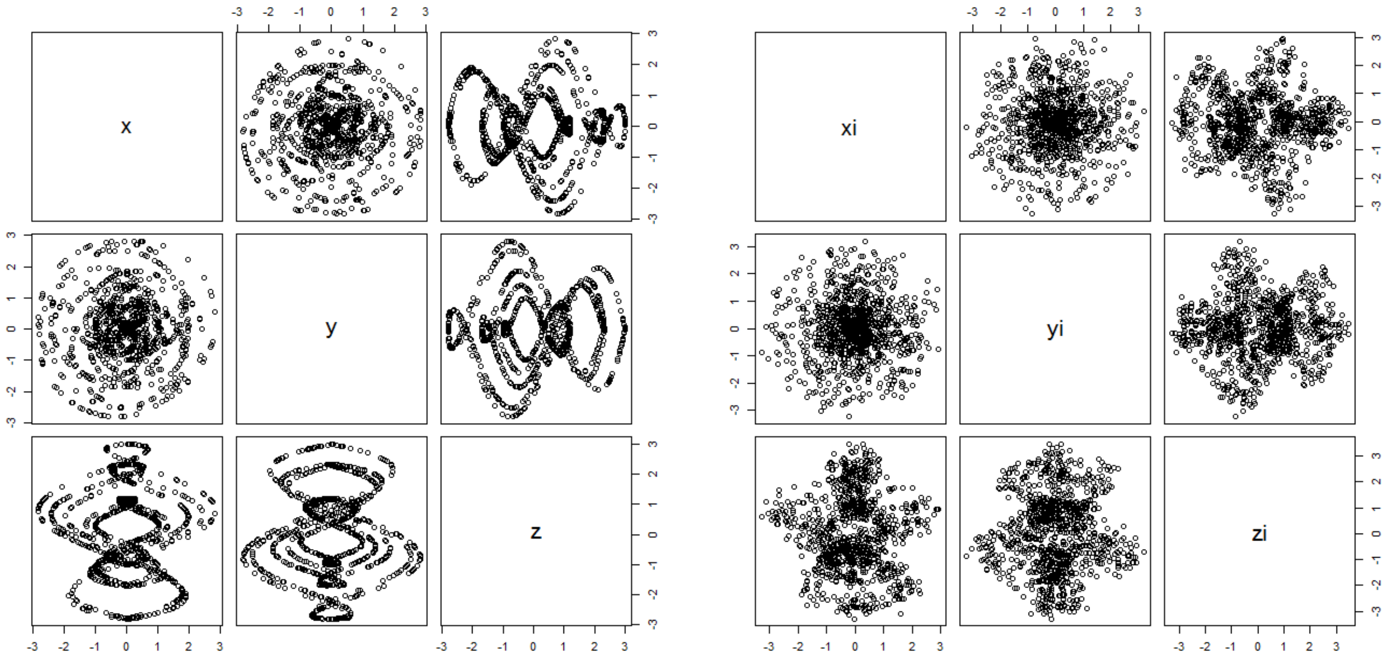

Figure 7. Scatterplot of relationship without random error and the observed values with random error.

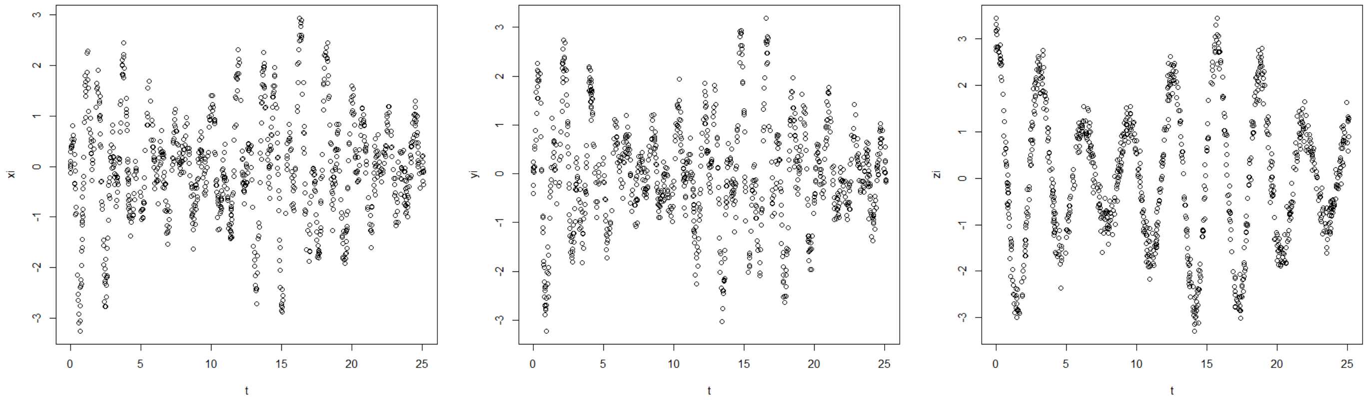

Figure 8. Line graph of measured variables.

$$t \in (0, 8\pi)$$
$$r(t) = 2 + \cos(0.4t)$$
$$x = r(t)\cos(5t)\sin(2t)$$
$$y = r(t)\sin(5t)\sin(2t)$$
$$z = r\cos(2t)$$
$$\delta \sim N(0, \sigma^2)$$
$$\varepsilon \sim N(0, \sigma^2)$$
$$\gamma \sim N(0, \sigma^2)$$
$$x_i = x + \delta_i$$
$$y_i = y_i + \varepsilon$$
$$z_i = z + \gamma_i$$

Modeling the spherical relationship given by

$$1 = \alpha_1 x + \alpha_2 y + \alpha_3 z + \alpha_4 xy + \alpha_5 xz + \alpha_6 yz + \alpha_7 x^2 + \alpha_8 y^2 + \alpha_9 z^2$$

The coefficient of determination is approximately 0.7 which is a measure of the constant nature of the system and indicates that the constant that balances the system is highly variant with nearly a uniform distribution.

Using the developed model to estimate unity (the constant the balances the system), we see the ossilation is more clearly defined, **(Figure 9)** and from which the frequency of ossilation can be determined in addition to the range. In this simulation the range is two.

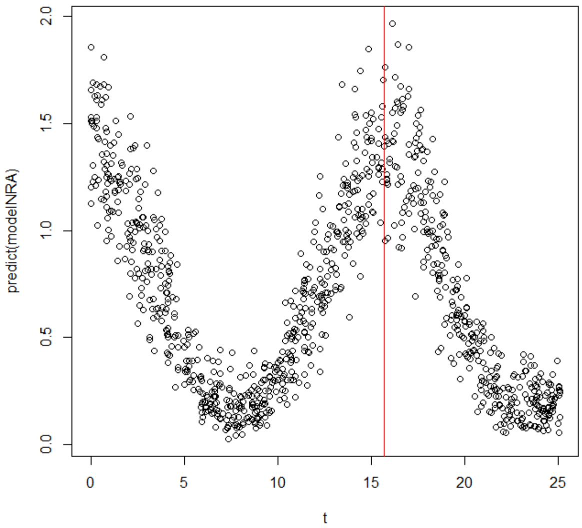

Figure 9. Time series of the predicted value of unity with the period of 5π marked in red.

Furthermore, as each measured variable is expected to be zero, by considering the reduced model given by $1=\alpha_1 x^2+\alpha_2 y^2+\alpha_3 z^2$,

Using the developed model, $u=\hat{1}=a_1 x^2+a_2 y^2+a_3 z^2$ on average, we can estimate the mean radius using $\overline{u}$, we have, $\hat{r}_x=\frac{\overline{u}}{\alpha_1}, \hat{r}_y=\frac{\overline{u}}{\alpha_2}$, and $\hat{r}_z=\frac{\overline{u}}{\alpha_3}$. In this simulation the radius was approximately two for each estimate of the radius.

CONCLUSION

In this review, analysis shows that non-response model allows for information about unmeasured variables to be detected and with proper modeling, estimated. Hence, non-response analysis can measure the constant nature of a single variable; measure the balancing constant in a stable system and estimate the subject response in an evolving system.

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