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Parametric Analysis of Carbon Dioxide in the Atmosphere

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Abstract: Two important entities that constitute global warming are atmospheric temperature and carbon dioxide (CO_2). The present study is to use actual CO_2 data from the various locations including Hawaii and Alaska and identify the actual probability distribution functions (pdf) that probabilistically characterize its behavior. Having such a pdf of CO_2 we can proceed to perform parametric statistical analysis and obtain needed useful information. Presently, scientists working on the subject matter of CO_2 characterize the pdf as the classical and popular Gaussian pdf. We have found that the three parameter Weibull pdf gives a much better fit to CO_2 and the Gaussian is statistically rejected. In addition, we perform trend analysis and identify that the behavior of CO_2 as a function of time is quadratic. We proceed to filter the data accordingly to be independent of time and the subject data follows a general logistic pdf. Utilizing this finding we proceed to obtain ten and twenty year projections of CO_2 in the atmosphere along with appropriate degrees of confidence.

Key words: Parametric analysis, trend analysis, profiling, return periods, atmospheric carbon dioxide

INTRODUCTION

Global warming is a significant event that needs continuous monitoring in order to have a better understanding of the phenomenon, both cause and effect. Many argue that the observed increase in the average atmospheric and oceanic temperatures (Gerber, 1991) will continue to increase over time (Hackett and Tsokos, 2009) and others believe that this is just a natural high, that the earth's lifeline is significantly longer than human records. That is, oscillation that occur every millennium cannot be detected with data only spanning decades or even centuries, of recorded history. As shown in Fig. 1, the schematic view of the amount of carbon dioxide in the atmosphere is rather complex.

Some cause of warming is the increased atmospheric concentrations of greenhouse gases including water vapor, carbon dioxide, methane, nitrous oxide and ozone. Atmospheric carbon dioxide (CO_2) and atmospheric temperature (T) are two important variables related (indirectly/non-linear). The objective of the present study is to probabilistically determine the best probability distribution that characterizes the behavior of CO_2. Presently scientists working in the subject area make the assumption that CO_2 in the atmosphere follows the classical Gaussian probability distribution and that is not the best possible probabilistic characterization for decision making.

It will be shown in the present study that the best-fit probability distribution function that probabilistically characterizes the CO_2 data in the atmosphere follows the general logistic pdf. Having identified statistically the best fit probability distribution of the subject data, it allows scientists to determine confidence limits, testing of hypothesis, to estimate the key parameter (expectation, variance, etc.) that are useful in understanding the expected behavior of CO_2, return periods, among others. For the pros and cons of global warming (Gerber, 1991; Hackett and Tsokos, 2009; Shih and Tsokos, 2008a-c, 2009; Xu and Tsokos, 2008, 2009).

CARBON DIOXIDE DATA

The information that we shall use in the present study consists of several data sets. The data of primary interest is atmospheric carbon dioxide in the air recorded at several sites located at various latitudes in the open water in the Pacific Ocean including the Hawaiian Islands (Mauna Loa), shown in Fig. 2. The data was gathered and maintained by Scripps Institution of Oceanography. Monthly values are recorded and are expressed in parts per million (ppm).

Figure 3 compares the carbon dioxide measurements for the various locations shown in Fig. 2 for the years 1990 to 2004. The locations are alphabetically coded in order from North to South. This shows that the further

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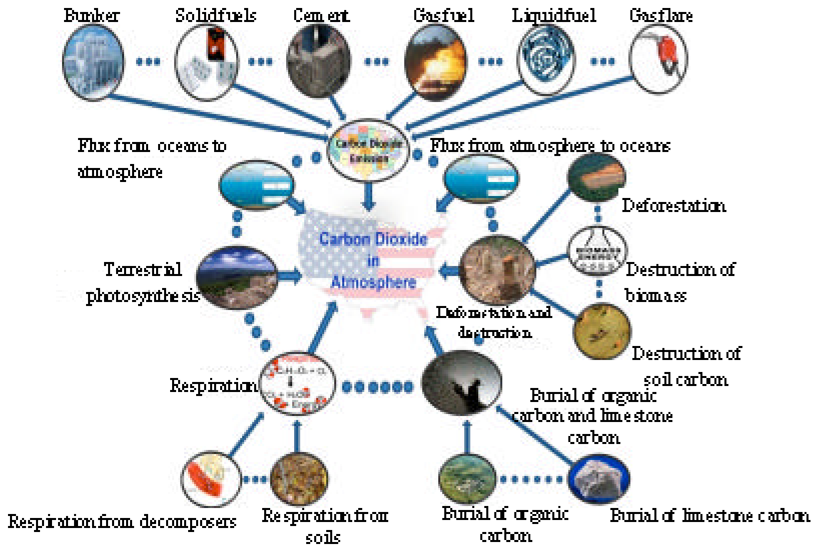

Fig. 1: Schematic view of carbon dioxide in the atmosphere in USA

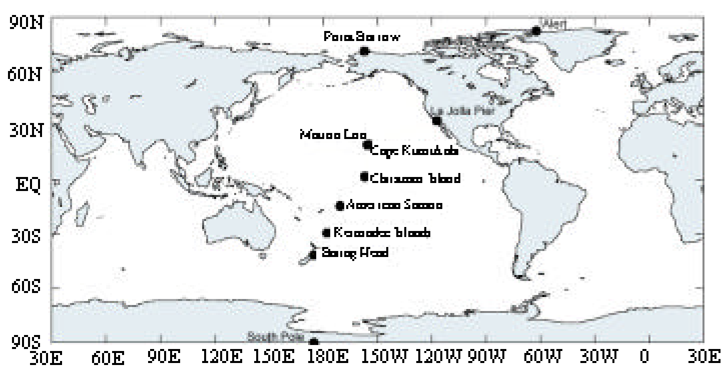

Fig. 2: Map of monitoring sites for carbon dioxide in the atmosphere

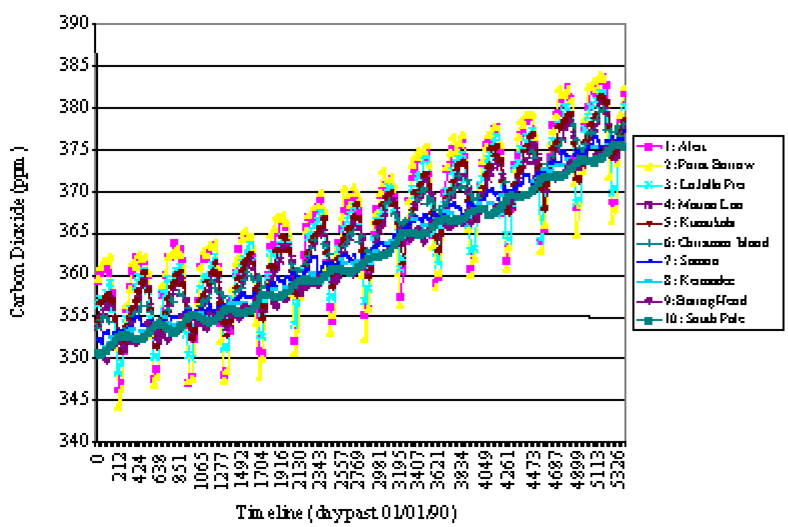

Fig. 3: Line graph of atmospheric carbon dioxide (ppm) by location

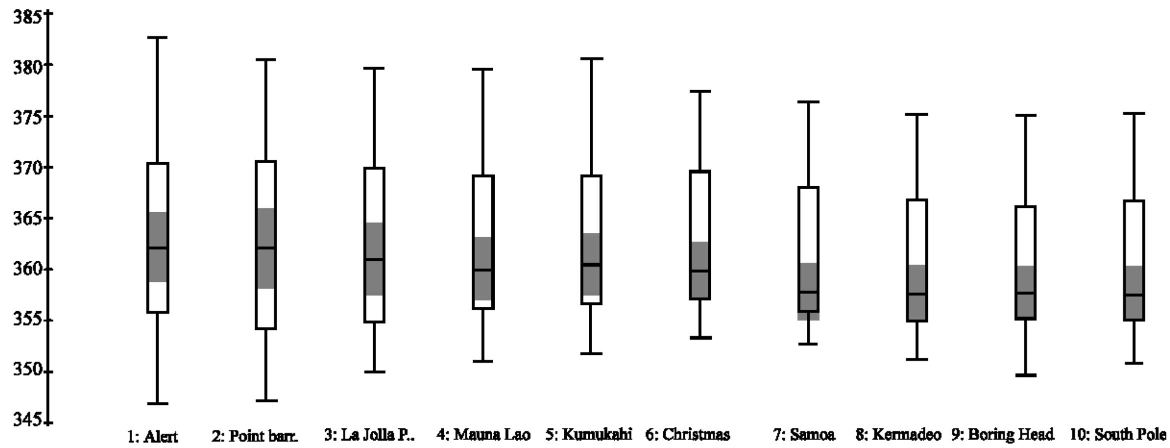

Fig. 4: Box plots of carbon dioxide in the atmosphere by location from South to North

Table 1: Summary of basic statistics for atmospheric carbon dioxide by location

Location	Count	Mean carbon dioxide	Median carbon dioxide	SD
Alert	179	365.916	364.94	8.906
Point Barrow	176	366.047	365.40	9.293
La Jolla Pier	173	365.106	364.50	8.106
Mauna Loa	180	364.728	364.38	7.602
Kumukahi	180	365.148	364.47	7.76
Christmas Island	126	364.541	363.84	7.698
Samoa	180	363.433	362.28	7.298
Kermadec	98	361.985	361.09	7.135
Baring Head	117	360.352	358.86	6.905
South Pole	178	362.247	361.07	7.236

North, the more variability in the amounts of carbon dioxide (ppm) and the lower variability in the South. However, all these locations are highly correlated with the smallest coefficient of correlation being 0.73. All locations show an increase in atmospheric carbon dioxide (ppm) over the years.

Furthermore, the maximum yearly concentrations occur every May and the minimum yearly concentrations occur in October. Box plots of the amount of CO_2 in the atmosphere (ppm) for the various locations from A to J as identified in Fig. 3 is shown by Fig. 4, which support the fact that the mean and variance in the amount of CO_2 in the atmosphere depends on the location. From the North Pole moving toward the South Pole, the further North readings are made, the more consistent the CO_2 readings. This is also clearly shown by Fig. 3; while all stations show variability in a way of a trend and seasonal effects, the fluctuation over time is significantly greater further North. Moreover, the variability shown in Fig. 4 is more likely due to trending.

The summary of the basic descriptive statistics for all the locations that collect data regarding CO_2 in atmosphere is shows in Table 1. It can be seen that the sample mean and standard deviation are quite homogeneous.

Table 2: Summary statistics for atmospheric carbon dioxide-all locations

Statistic	Values
Sample size	1587
Range	39.62
Mean	364.17
Variance	64.834
SD	8.052
Coef. of variation	0.02211
SE	0.20212
Skewness	0.19848
Kurtosis	-0.8238

When considered collectively, we obtain the basic descriptive statistics summarized in Table 2. The sample mean is quite similar among the stations (Table 1).

PARAMETRIC ANALYSIS

Scientists working with CO_2 data as measured in the atmosphere assume that it follows the classical Gaussian probability distribution. We shall first show that this is not a correct assumption which will lead to misunderstanding the behavior of the subject data. Secondly, we shall statistically identify that the best fit probability distribution function (pdf) that characterizes the CO_2 data is the three-parameter Weibull pdf.

The CO_2 data from Mauna Loa has a sample mean of 364.728 ppm with a sample standard deviation of 7.602. All locations considered collectively have a sample mean of 364.165 ppm and a sample standard deviation of 8.052. Hence, on the average there is no significant difference between the amount of carbon dioxide measured in Mauna Loa, Hawaii and all ten locations combined, however, the further South, the greater the variance; moreover, the greater the sample mean, the greater the sample standard deviation. However, we can statistically conclude that there is no significant difference between these two (true) means.

Table 3: Goodness-of-Fit Tests (all stations)

	Tests		
Rank	Kolmogorov Smirnov	Anderson Darling	Chi-squared
1	Weibull (3P)	Weibull (3P)	Gen. extreme value
2	Gen. extreme value	Gen. extreme value	Weibull (3P)

Table 4: Goodness-of-Fit Tests (Mauna Loa, Hawaii)

	Tests		
Rank	Kolmogorov Smirnov	Anderson Darling	Chi-squared
1	Weibull(3P)	Gen. extreme value	Gen. extreme value
2	Gen. extreme value	Weibull (3P)	Weibull (3P)

Furthermore, this data does not display symmetry nor show bell-shaped smoothness, an indication of being normally distributed. Furthermore, the normal probability plot supports that the subject data's distribution is not Gaussian.

Under the assumption that the probability distribution is unbounded, that is, the probability distribution is not characterized by a uniform, beta, triangular pdfs, etc. but may be characterized by one of following pdf: Exponential, Frechet (3P), Gen. extreme value, Gen. logistic, Gumbel max, Gumbel min, Inv. Gaussian, Log-logistic, Logistic, Lognormal (3P), Normal, Pareto and Weibull.

In searching for the best possible probability distribution that characterizes the behavior of CO_2, we tested twenty-seven different well defined probability distributions using three standard statistical tests: Kolmogorov Smirnov Anderson-Darling and Chi-squared. The first test, Kolmogorov Smirnov, is based on minimum difference estimation. Anderson-Darling measures whether the data can be transformed into the uniform probability distribution. The Chi-square test for goodness-of-fit is a measure of relative error squared. Using these three measures, the Gaussian pdf did not pass any of the tests-the Gaussian pdf does not rank in the top five. Moreover, the three-parameter Weibull pdf which ranks in the top two in all tests is a specific form of the General Extreme Value pdf which also ranks in the top two for all tests for goodness-of-fit (Table 3, 4).

Thus, we can conclude that the three-parameter Weibull pdf best characterizes the probability distribution of the amount of carbon dioxide in the atmosphere and its pdf is given by:

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \exp\left\{-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right\}; \alpha > 0, \beta > 0, \gamma < x < \infty \quad (1)$$

where, γ is the location, β is the scale and α is shape parameters, respectively.

The approximate Maximum Likelihood Estimates (MLE) of the parameters α, β and γ given in Eq. 1 are given by Table 5.

Table 5: Approximate MLE of α, β and γ

Data source	Parameter estimates
All stations	$\hat{\alpha} = 2.779, \hat{\beta} = 23.029, \hat{\gamma} = 343.7$
Mauna Loa, Hawaii	$\hat{\alpha} = 2.108, \hat{\beta} = 17.092, \hat{\gamma} = 349.6$

Table 6: Confidence Limits of the True Mean of CO_2 for all stations and Mauna Loa

	All		Mauna Loa, Hawaii	
Level (%)	Lower limit	Upper limit	Lower limit	Upper limit
99	347.125	385.6614	350.9859	387.2974
95	349.8342	380.5356	352.5882	381.3481
90	351.6086	377.8775	353.7770	378.3632

The confidence limits for all stations and Mauna Loa, Hawaii are not statistically different and hence the remainder of this study will concentrate on all stations collectively

Thus, the Weibull pdf for all stations is given by:

$$f(x) = 0.1119\left(\frac{x-343.7}{23.029}\right)^{1.779} \exp\left\{-\left(\frac{x-343.7}{23.029}\right)^{2.779}\right\}, \quad (2)$$

and for Mauna Loa, Hawaii is given by:

$$f(x) = 0.0648\left(\frac{x-349.6}{17.092}\right)^{1.108} \exp\left\{-\left(\frac{x-349.6}{17.092}\right)^{2.108}\right\} \quad (3)$$

The cumulative probability distribution for all the stations is given by:

$$F(x) = 1-\exp\left\{-\left(\frac{x-343.7}{23.029}\right)^{2.779}\right\}, \quad (4)$$

and for the station located in Mauna Loa, Hawaii, is given by:

$$F(x) = 1-\exp\left\{-\left(\frac{x-349.6}{17.092}\right)^{2.108}\right\} \quad (5)$$

A graphical view of Eq. 2 and 3 is given by Fig. 5 and similarly the graph of Eq. 4 and 5 are shown in Fig. 6.

The above graphs show that the data considered collectively is similar in distribution to the individual station located in Mauna Loa, Hawaii. The probability distributions are slightly skewed and this skewness is more clearly illustrated at the individual station in Hawaii.

Interval estimates at the 90, 95 and 99% level of confidence are given for the true mean amount of CO_2 in the atmosphere for all stations and Hawaii based on the three-parameter Weibull PDF are shown in Table 6.

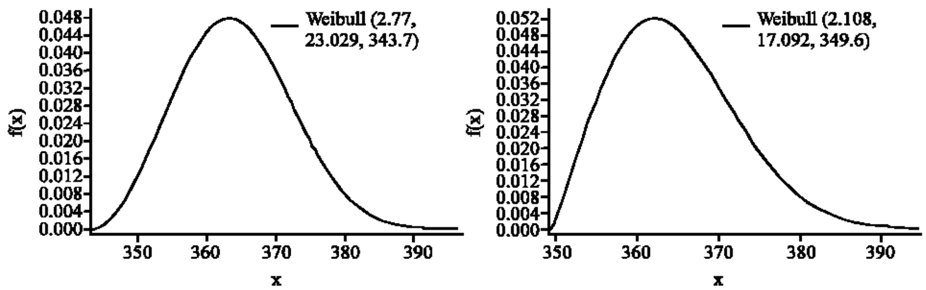

Fig. 5: Probability density functions

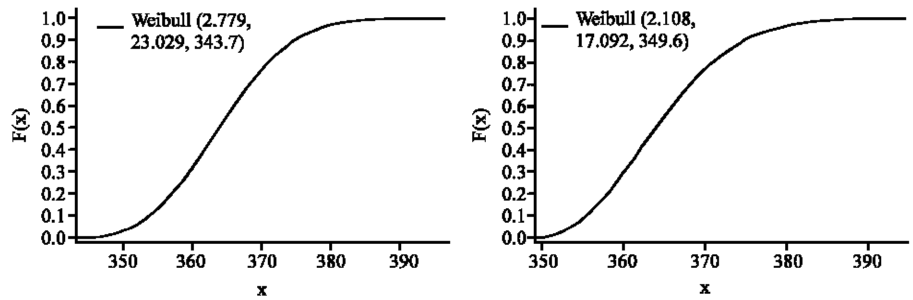

Fig. 6: Cumulative distribution functions

TREND ANALYSIS

The mean amount of CO_2 present in the atmosphere has increased over time. In part, this may be due to the human population-regardless, both tend to vary with time. Understanding these trends would be useful information in projecting how much CO_2 is present in the atmosphere, not simply as a constant, but as a function of time.

Consider the distributional behavior of CO_2 over the 14 year period as shown in Fig. 7. Here, we see that the mean of CO_2 measurements in the atmosphere is definitely not constant with respect to time. That is, there is a steady increase in the amount of carbon dioxide measured in the atmosphere over time. This trend is usually assumed to be constant; however, there appears to be either a linear, quadratic or exponential trend. A linear trend would indicate a steady increase in the mean amount of carbon dioxide in the atmosphere whereas a quadratic trend would indicate not only that there is an increase in the mean, but that the rate of increase is accelerating. The exponential trend also allows for the rate to have an exponential growth with respect to time.

Linear relationship: Here, we consider the average increase in the Atmospheric Carbon Dioxide (ACD) varies linearly with respect to time. That is, it can be stated as a simple statistical regression model, $x(t) = \beta_0+\beta_1 t+\varepsilon$, where, the coefficients βs are the weights that drive the subject response and ε is the random error. This model assumes the rate at which the yearly mean CO_2 is introduced into the atmosphere is constant, $dx/dt = \beta_1$ and no acceleration exists, $d^2x/dt^2 = 0$.

Using the CO_2 data from all stations, the estimated regression model is given by:

$$\hat{x}(t) = 351.964 + 0.004553t \quad (6)$$

The developed statistical model, Eq. 6 has an R^2 value of 0.779; that is, it explains 79.9% of the variation in the subject response and yields $d\hat{x}/dt = 0.004553$, an increase in carbon dioxide in the atmosphere over a 14-year period, from 1990 to 2004, of only 0.004553 ppm per day. This indicates an annual increase of 1.66 ppm per year and approximately 16.6 ppm per decade and this increase has persisted for five decades with no signs of slowing.

Quadratic relationship: Moreover, the residuals shown in Fig. 8 indicate that there is curvature in the relationship of CO_2 and time, that is, $d^2x/dt^2 \neq 0$. In fact, $d^2x/dt^2 > 0$, which means that the rate at which CO_2 is entering the atmosphere is increasing with respect to time. Hence, we will consider the second order model with respect to time to account for this variable rate.

Consider the second-order regression model, which allows for acceleration in the amount of CO_2 in the atmosphere; that is:

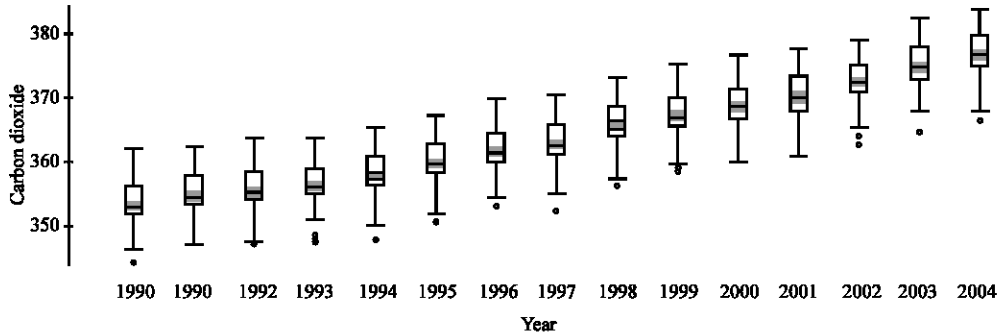

Fig. 7: Box plot of atmospheric carbon dioxide for all stations by year

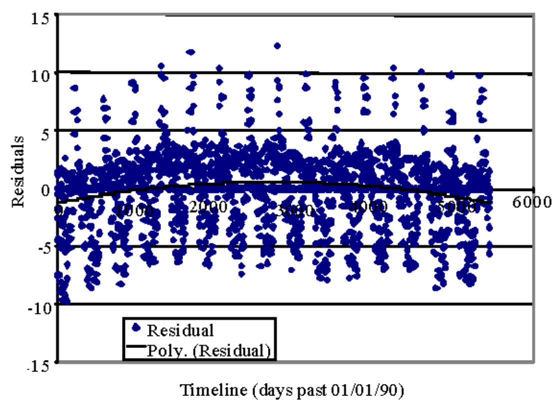

Fig. 8: Residual plot for the developed linear model

$$x(t) = \beta_0+\beta_1 t+\beta_2 t^2+\varepsilon,$$

where, the coefficients βs are the weights that drive the subject response and ε is the random error. Using the data from all the stations, the statistical model is estimated by:

$$\hat{x}(t) = 353.288 + 0.0030699t + 2.73729\times10^{-7}t^2 \quad (7)$$

The developed statistical model, Eq. 7, explains 80.5% of the variation in the subject response. This yields $d\hat{x}/dt = 0.00307 + 5.48\times10^{-7}t$ and $d^2\hat{x}/dt^2 = 5.48\times10^{-7}$, that is, there has been positive increase in the rate at which carbon dioxide is accumulating in the atmosphere. Furthermore, even though this acceleration is extremely small, this is the daily acceleration over decades.

Exponential relationship: Consider the exponential relation, which allows for CO_2 to have an exponential growth rate. The model is given by:

$$x(t) = ae^{bt} +\varepsilon$$

where, a and b are the parameters that drive the estimation of the response and ε is the random error. Assuming that E(ε) = 0, the above statistical model can be written as:

Table 7: Coefficient of determination and Chi-square test statistic for goodness of fit

Statistical model	R-squared (%)	Chi-squared
Linear	79.9	56.64
Quadratic	80.5	54.98
Exponential	80.0	56.32

$$\ln\hat{x}(t) = \ln\hat{a} + \hat{b}t.$$

Using actual CO_2 data, we find $\hat{a} = 352.0982$ and $\hat{b} = 1.25\times10^{-5}$, which yields an initial rate of increase, $d\hat{x}/dt = 0.0044$, when t = 0, to $d\hat{x}/dt = 0.00552$, after 50 years. However, if we compare this exponential model with the linear and quadratic model as show by Fig. 9, we see that the quadratic best fits the data except for the seasonality. The values of the coefficient of determination R^2 and the goodness-of-fit statistic χ^2 further demonstrate this point, both shown in Table 7.

Comparison of trend analysis: In Fig. 10 a graphical comparison of the linear, quadratic and exponential trends as a function of time.

Graphically, all three address the issue of the monotonically increase trend. However, as there is curvature the linear equation will pull away from the data as time progresses. Moreover, using the coefficient of determination, R^2 and the chi-squared measure of relative error, the quadratic trend is the best fit statistical model.

Hence, given the developed statistical model, we can determine the probability distribution of CO_2 independent of time by filtering the data using the quadratic function (Fig. 11). Hence, consider the residual data that remains once the trend has been removed given by:

$$\hat{y}(t) = \hat{x}(t) - \beta_1 t - \beta_2 t^2 \cdot$$

We proceed to search for a pdf that best fits the data after removing the quadratic trends. We investigate a number of pdf including the Generalized Extreme Value (GEV) family of pdfs and we can conclude the generalized logistic pdf gives the best probabilistic characterization of the filtered data over the GEV pdf that came second before (Fig. 12).

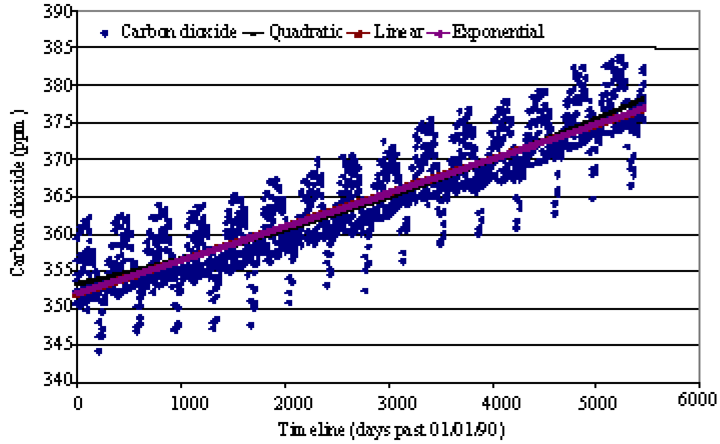

Fig. 9: Line graph comparing the ACD data and the following statistical models: quadratic, linear and exponential

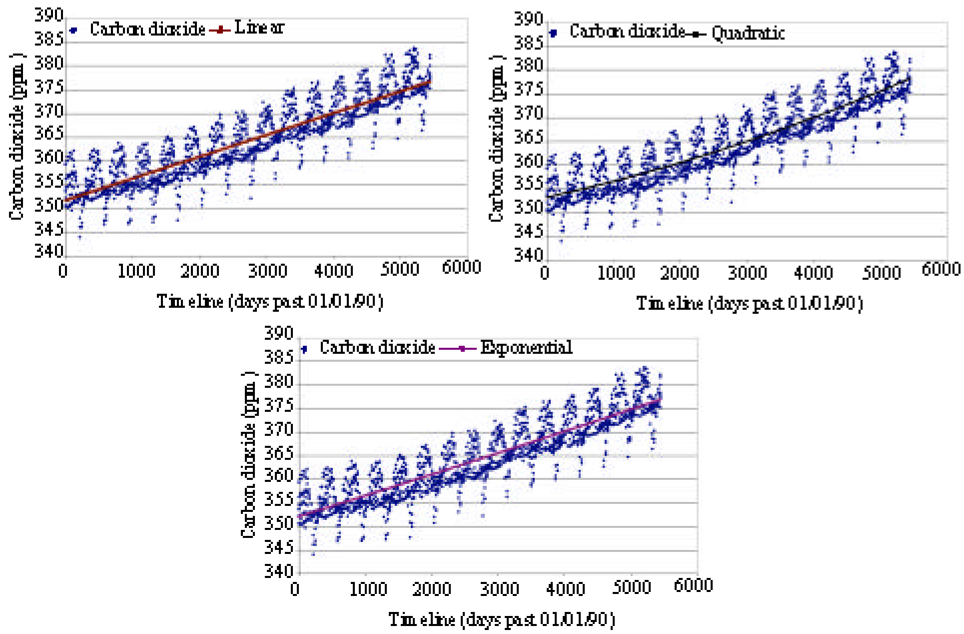

Fig. 10: Line graph comparing the ACD data and the following statistical models: quadratic, linear and exponential

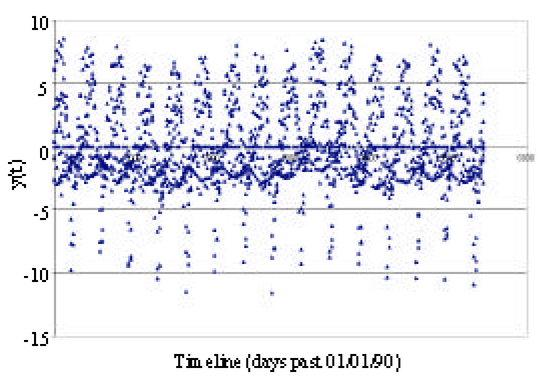

Fig. 11: Line graph of filtered data $\hat{y}(t) = \hat{x}(t) - \beta_1 t - \beta_2 t^2$

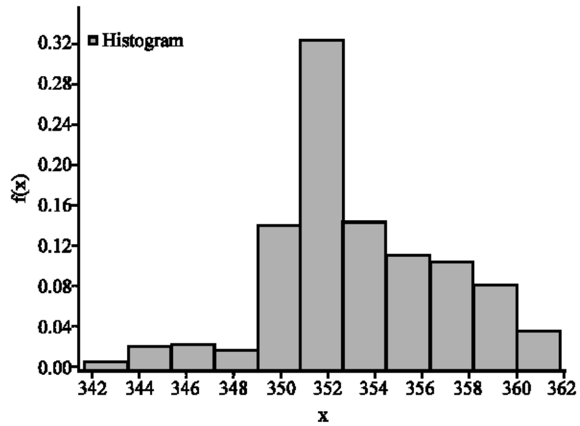

Fig. 12: Histogram of data independent of time (filtered data)

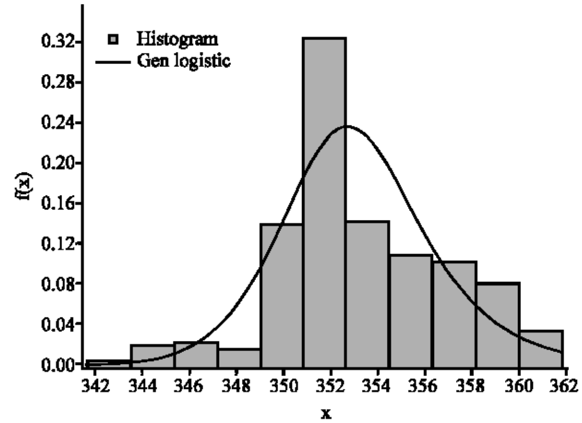

Fig. 13: General logistic pdf a top the histogram of data independent of time

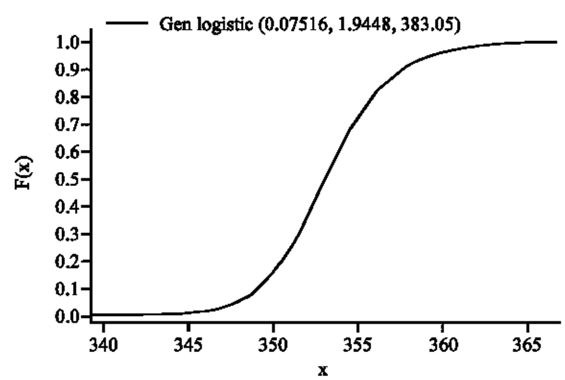

Fig. 14: General logistic cdf of data independent of time

PARAMETRIC ANALYSIS OF FILTERED DATA

Table 8 supports the pdf choice.

The general logistic pdf is given by:

$$f(y)=\frac{(1+\xi z)}{\sigma\left[1+(1+\xi z)^{-1/\xi}\right]^2}, z=\frac{y-\mu}{\sigma}, 1+\xi\frac{(y-\mu)}{\sigma}>0 \qquad (8)$$

where, μ is the location, σ is the scale and ξ is the shape parameters, respectively. The cumulative pdf of Eq. 8 is given by:

$$F(y)=\frac{1}{1+(1+\xi z)^{-1/\xi}}$$

The maximum likelihood estimates of the parameters are $\hat{\xi}=0.07516$, $\hat{\mu}=353.05$ and $\hat{\sigma}=1.9448$. Thus, the general logistic pdf for the stations overall is given by:

$$f(y)=\frac{(1+0.07516z)}{1.9448\left[1+(1+0.07516z)^{-1/\xi}\right]^2}, \qquad (9)$$

where, $z=\frac{y-353.05}{1.9448}$ for $1+0.07516\frac{(y-353.05)}{1.9448}>0$.

The corresponding cumulative probability distribution is given by:

$$F(y)=\frac{1}{1+(1+0.07516z)^{-1/0.07516}}. \qquad (10)$$

A graphical view of Eq. 9 and 10 is given by Fig. 13 and 14, respectively.

We shall use the cumulative probability distribution which is shown above to generate projected rates of CO_2 in the atmosphere.

Table 8: Goodness-of-Fit Test (all stations)

Rank	Kolmogorov Smirnov	Anderson Darling	Chi-Squared
1	General Logistic	General Logistic	Gen. Ext. Value
2	Gen. Ext. Value	Log-Logistic (3P)	General Logistic

Table 9: Confidence Intervals for true mean of CO_2 for both unfiltered and filtered data

	Weibull (Unfiltered data)		General logistic (Filtered data)	
Level (%)	Lower limit	Lower limit	Lower limit	Upper limit
99	347.1250	385.6614	344.56	365.69
95	349.8342	380.5356	346.82	361.25
90	351.6086	377.8775	347.91	359.46

Using the General logistic pdf we obtain 90, 95 and 99% confidence intervals for the true mean amount of CO_2 in the atmosphere for the filtered data for all stations. The results are shown in Table 9, along with the similar confidence intervals for the time dependent data.

To compare, for the unfiltered data, the confidence interval for the amount of CO_2 in the atmosphere with level of significance α is given by:

$$F^{-1}(\alpha/2)<x<F^{-1}(1-\alpha/2)$$

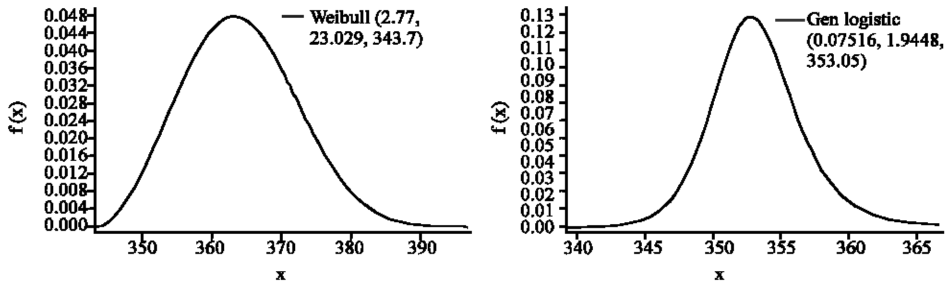

Fig. 15: Comparison of the three-parameter Weibull pdf for the non-filtered data and the general logistic pdf for the filtered data

where, $F(x) = 1 - \exp\left\{-\left(\frac{x-343.7}{23.029}\right)^{2.779}\right\}$

For the filtered data recall, $\hat{y}(t) = \hat{x}(t) - \beta_1 t - \beta_2 t^2$ and solving for $\hat{x}(t)$, we have $\hat{x}(t) = \hat{y}(t) + \beta_1 t + \beta_2 t^2$ and hence the confidence interval for the amount of CO_2 in the atmosphere with level of significance α is given by:

$$F^{-1}(\alpha/2) + 0.003699t + 2.73729\times10^{-7}t^2 < x(t) < F^{-1}(1-\alpha/2) + 0.003699t + 2.73729\times10^{-7}t^2$$

where, $F(y) = \frac{1}{1+(1+0.07516z)^{-1/0.07516}}$ and $z = \frac{y-353.05}{1.9448}$

Comparing these interval estimates with those found using the three-parameter Weibull, the general logistic pdf gives slightly smaller lower limits; for the 99% confidence intervals for unfiltered data, the three-parameter Weibull has a lower limit of 347.125 ppm whereas for the filtered data, the General Logistic has a lower limit of 344.56 ppm, a difference of about 3 ppm (Fig. 15). However, there is a significant difference in the upper limits as the increase in CO_2 in the atmosphere over time which appears as a skew in the data when unfiltered and assumed independent of time. Whereas the three-parameter Weibull estimates an upper bound of 385.6614 ppm, without the increase in CO_2 over time, the General Logistic estimates an upper bound of 365.69 ppm.

PROJECTIONS

We can utilize the cumulative pdf given by Eq. 11 that we have identified to obtain future projection of CO_2 in the atmosphere with an appropriate degree of confidence, Fig. 16.

The cumulative pdf for all the stations and filtered data is given by:

$$F(y) = \frac{1}{1+\left(1+0.07516\left(\frac{y-353.05}{1.9448}\right)\right)^{-1/0.07516}} \qquad (11)$$

where, $y(t) = x(t) - \beta_1 t - \beta_1 t^2$.

Table 10: Confidence limits for the 90, 95 and 99% confidence intervals as a function of time

	90% CI		95% CI		99% CI	
Year	Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit
2009	382.4	393.95	381.31	395.74	379.05	400.18
2019	411.14	422.69	410.05	424.48	407.78	428.92
2020	414.41	425.96	413.32	427.75	411.06	432.19
2021	417.77	429.31	416.68	431.11	414.41	435.55
2022	421.19	432.73	420.1	434.53	417.83	438.97
2023	424.68	436.23	423.59	438.02	421.32	442.46
2024	428.25	439.79	427.15	441.59	424.89	446.03
2025	431.89	443.44	430.8	445.23	428.54	449.67
2026	435.61	447.15	434.51	448.95	432.25	453.39
2027	439.39	450.94	438.3	452.73	436.03	457.17
2028	443.25	454.79	442.16	456.59	439.89	461.03
2029	447.19	458.74	446.1	460.53	443.83	464.97

Projecting into the future ten years (to 2019), at a 95% level of confidence, we have that the probable amount of carbon dioxide in the atmosphere will be between 410.05 to 424.48 ppm, as shown by Fig. 17. Twenty years into the future (to 2029) at a 95% level of confidence, we have that the probable amount of carbon dioxide in the atmosphere will be between 446.10 and 460.53 ppm. Projecting fifty years into the future (to 2059) at a 95% level of confidence, we have that the probable amount of carbon dioxide in the atmosphere will be between 598.05 and 612.48 ppm, as shown by Fig. 18.

Note that from an estimated low CO_2 in the atmosphere of 379.05 ppm in 2009 to a mean of 443.83 ppm in twenty years (2029), this is an estimated 17% increase. However, in perspective, carbon dioxide is less than 1% of the atmosphere. If 379.05 ppm represents 1% of the atmosphere and the atmosphere is constant, then 443.83 ppm is 1.17% of the atmosphere; this is only a 0.17% increase. However, carbon dioxide is more like 0.038% of the atmosphere (Williams, 2009). Hence, if 379.05 ppm represents 0.038% of the atmosphere and the atmosphere is constant, then 443.83 ppm is 0.04449% of the atmosphere; this is only a 0.00649% increase. Confidence intervals at the 90, 95 and 99% level of confidence for the expected amount of carbon dioxide in the atmosphere on the first of each year is given in Table 10.

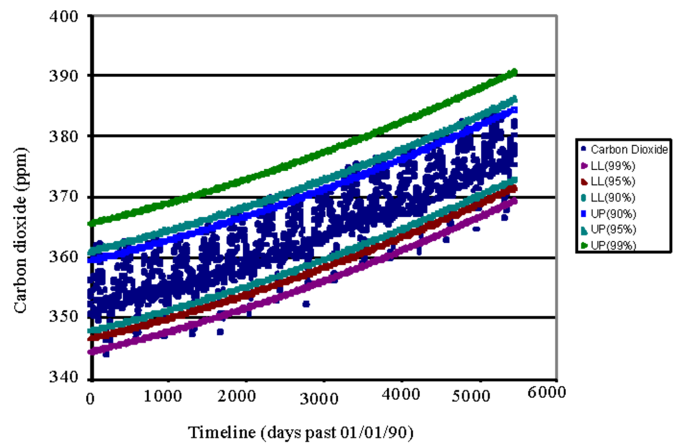

Fig. 16: Confidence intervals for 1990 to 2000

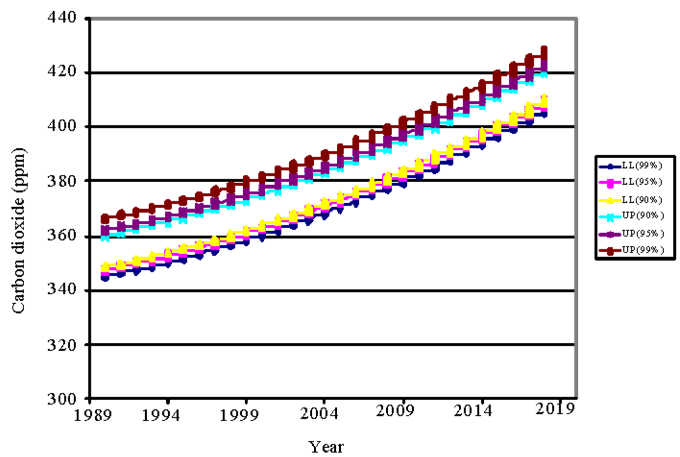

Fig. 17: Projections through 2019

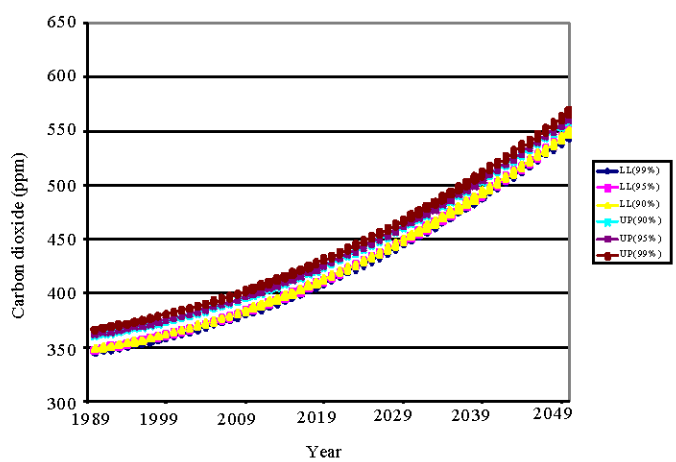

Fig. 18: Projections through 2059

The actual information that resulted in Fig. 17 and 18, is given by Table 10.

CONCLUSION

The amount of carbon dioxide in the atmosphere, if unfiltered and assumed independent of time, is best characterized by the three-parameter Weibull; however, the mean amount of carbon dioxide in the atmosphere has increased at an accelerated rate, statistically modeled by a quadratic function of time. The filtered information, which is independent of time, is best characterized by the general logistic probability distribution. Using the developed model, we can estimate yearly averages in 2009 of 389.9 ppm to an average of 354.8 ppm in 2009, approximately 10%. In the next ten years (2019), we expect the yearly average to be 419.0 ppm, a 7% increase over 2009. In the next twenty years (2029), we expect the yearly average to be 455.4 ppm, approximately a 17% increase over 2009. However, in perspective, if the original estimated in 1997 is 0.038% of the atmosphere in total, the amount of carbon dioxide in the atmosphere expected to increase by 0.0277% in the next fifty years to 0.0657% of the atmosphere. This is assuming that there is no point of saturation. Assuming that the atmospheric volume is constant, these various elements which constitute the atmosphere might began breaking down or transforming into other compounds; for example, to compensate for an increase in carbon dioxide, there might be a regeneration of wetlands or forestation.

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